

Surname	Centre Number	Candidate Number
First name(s)		2



GCE A LEVEL

A420U10-1



THURSDAY, 26 MAY 2022 – AFTERNOON

PHYSICS – A level component 1

Newtonian Physics

2 hours 15 minutes

For Examiner's use only			
	Question	Maximum Mark	Mark Awarded
Section A	1.	11	
	2.	10	
	3.	13	
	4.	9	
	5.	17	
	6.	9	
	7.	11	
Section B	8.	20	
Total		100	

ADDITIONAL MATERIALS

In addition to this examination paper, you will require a calculator and a **Data Booklet**.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

This paper is in 2 sections, **A** and **B**.

Section **A**: 80 marks. Answer **all** questions. You are advised to spend about 1 hour 35 minutes on this section.

Section **B**: 20 marks. Comprehension. You are advised to spend about 40 minutes on this section.

The number of marks is given in brackets at the end of each question or part-question.

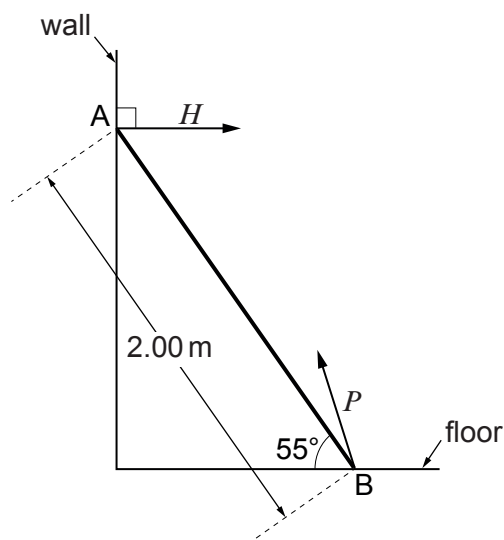
The assessment of the quality of extended response (QER) will take place in question **7(c)**.



JUN22A420U10101

SECTION AAnswer **all** questions.

1. A uniform plank, AB, of length 2.00 m and weight 232 N rests in equilibrium against a wall, at an angle of 55° to the horizontal, as shown in the side view. The wall exerts a horizontal force, H , on the plank. The floor is rough and exerts a force, P , on the plank.



- (a) (i) **Show the plank's weight on the diagram** using a correctly placed arrow. [1]

- (ii) Show that H is approximately 80 N, giving a brief explanation of your method. [4]

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- (iii) Write down the horizontal and vertical components of force P , and hence determine its magnitude and direction. [3]

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- (b) The maximum frictional force that the floor can exert on the plank is 90 N. Show that the plank could not be in equilibrium at an angle of 45° . [3]

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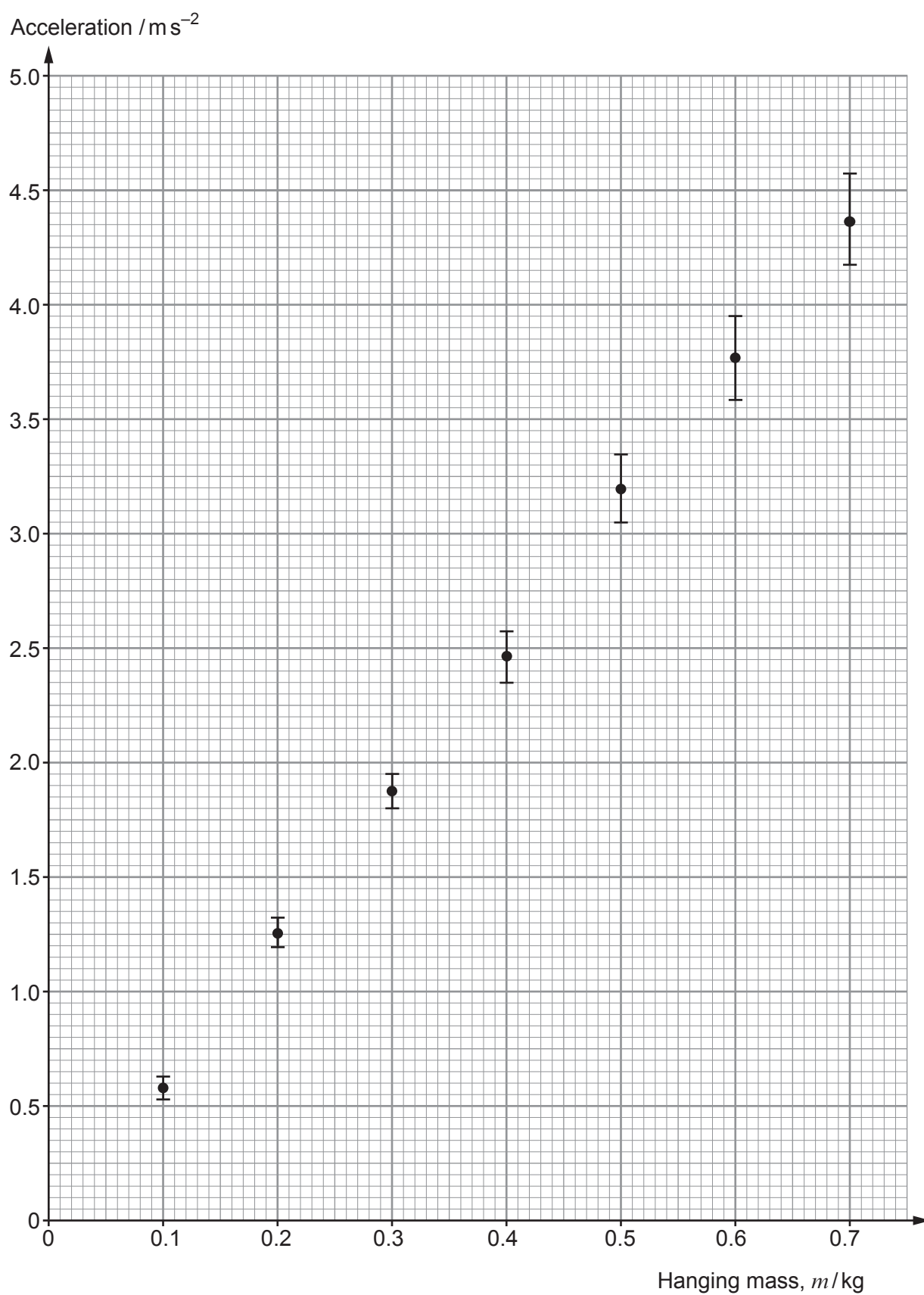
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- (c) Describe the measurements and calculations that Fatima might have made to determine the acceleration (for any one value of m). Do **not** consider uncertainties. [3]

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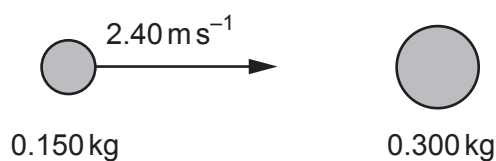


3. (a) (i) State the principle of conservation of momentum. [2]

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- (ii) A metal disc of mass 0.150 kg is sliding to the right at 2.40 m s^{-1} on a horizontal surface. It collides head-on with a disc of mass 0.300 kg which is initially stationary. See diagram.



View from above

After the collision the 0.300 kg disc has a velocity of 1.40 m s^{-1} **to the right**.

- I. Determine the speed and direction of the 0.150 kg disc after the collision. [3]

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- II. Calculate the kinetic energy lost in the collision. [2]

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- III. Explain how the principle of conservation of energy applies to this collision. [2]

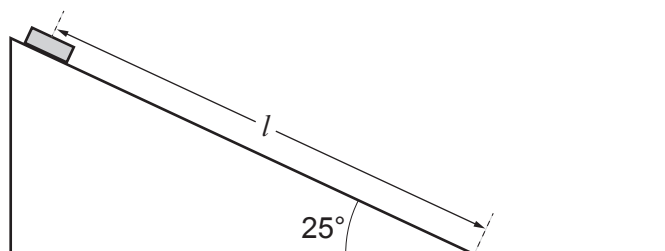
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- (b) Cedric wishes to investigate the collision experimentally. He plans to release the 0.150 kg disc from a distance, l , up a ramp. See diagram.



View from side

He calculates that l needs to be **greater than** approximately 0.7 m for the disc to reach a speed of 2.40 m s^{-1} by the bottom. Evaluate whether he is correct. [4]

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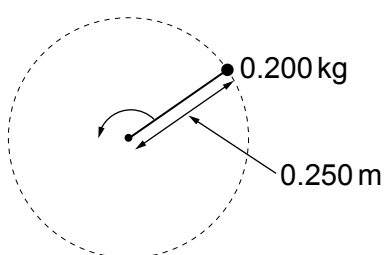
4. (a) Explain why an object moving in a circular path requires a resultant force to act on it, even when it is travelling at constant speed. [2]

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- (b) Behind a safety screen in an engineering laboratory, a metal sphere of mass 0.200 kg is whirled in a horizontal circle on the end of a thin steel rod.



View from above

- (i) The breaking stress of the steel under tension is 450 MPa and the **rod's diameter** is 1.2 mm . Show that the greatest force that the rod can exert is roughly 500 N . [2]

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- (ii) Calculate the greatest rotation frequency (number of revolutions per second) at which the sphere can be whirled before the rod breaks. [3]

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- (iii) I. State an assumption that you have made (or factor that you have ignored) in your calculation for part (b)(ii). [1]

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- II. Bearing in mind your previous answer, will the greatest rotation frequency before the rod breaks be larger or smaller than your calculated value? Justify your answer briefly. [1]

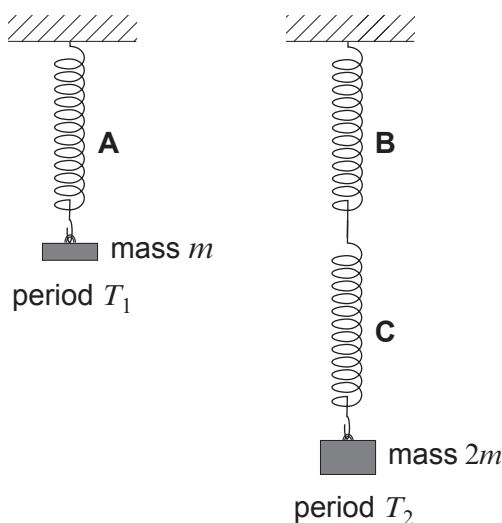
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5. (a) The three springs, **A**, **B**, **C**, shown in the diagrams are identical and of negligible mass. Each spring has spring constant k .



For the system on the left, the period of natural oscillations is T_1 . For the system on the right, the period is T_2 . Calculate the ratio $\frac{T_2}{T_1}$, giving your reasoning. [3]

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- (b) In a separate experiment a mass hanging from a spring is displaced **upwards** by 30 mm from its equilibrium position and released at time $t = 0$. It performs simple harmonic motion with a period of 1.20 s.

- (i) Show that the maximum velocity of the mass is approximately 0.16 m s^{-1} . [2]

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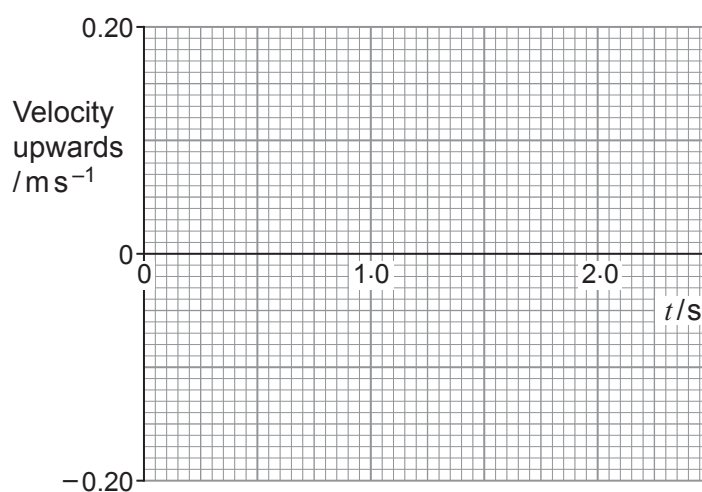
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- (ii) **Sketch** a velocity-time graph for the mass **on the grid provided**.

[2]

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- (iii) Calculate the speed of the mass at $t = 3.50$ s.

[2]

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- (iv) Ursula thinks that the kinetic energy of the mass varies at a frequency of 1.67 Hz. Evaluate this claim.

[2]

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- (c) Ursula modified the mass-spring system in part (b) so that its oscillations are significantly damped.

(i) Suggest what practical modification she might have made. [1]

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- (ii) Ursula measured the amplitude of the oscillations at equal time intervals. Here are her results:

Amplitude / mm	30	24	19	15
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Evaluate whether or not these results are consistent with an exponential decay of the amplitude. [2]

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- (d) The physics of damping has been used in designing car suspensions and suspension bridges for pedestrians. Compare the **purposes** that damping serves in these two cases. [3]

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6. (a) Define pressure.

[1]

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(b) A sealed canister contains 0.200 mol of oxygen (relative molecular mass: 32.0). An identical canister contains 0.200 mol of nitrogen (relative molecular mass: 28.0) at the same temperature. Giving your reasoning, determine these ratios:

(i) $\frac{\text{pressure of oxygen}}{\text{pressure of nitrogen}}$

[2]

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(ii) $\frac{\text{rms speed of oxygen molecules}}{\text{rms speed of nitrogen molecules}}$

[2]

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(c) (i) At the centre of the Sun the pressure is estimated to be 2.5×10^{16} Pa, and the density, $1.6 \times 10^5 \text{ kg m}^{-3}$. Calculate the rms speed of the particles in that region, treating them as molecules of an ideal gas.

[2]

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(ii) State **two** reasons why the ideal gas kinetic theory is not likely to give accurate results in this case.

[2]

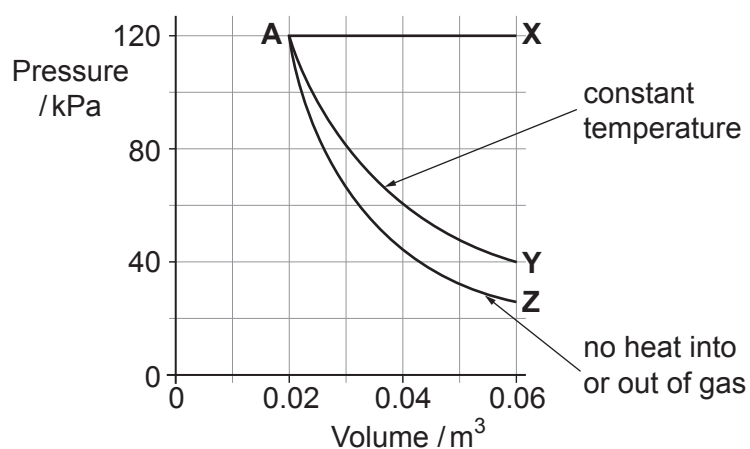
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7. The diagram shows three ways (AX, AY and AZ) in which 0.90 mol of an ideal gas can expand from a volume of 0.020 m^3 to a volume of 0.060 m^3 .



- (a) Calculate the temperatures at **A** and at **X**.

[3]

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- (b) Calculate the work done over **AX**.

[2]

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- (c) For both AY and AZ compare the work done, internal energy change and heat flow **with those for AX**. Calculations are not wanted, but you must give your reasoning. [6 QER]

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SECTION B

Answer **all** questions.

8. Read through the following article carefully.

Laser Doppler Cooling

Paragraph

Here, the laser and the humble Doppler shift combine to produce an incredibly cool effect, but what is Laser Doppler Cooling?

1

Perhaps the most obvious place to start is with the A level Doppler shift equation.

2

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = -\frac{\Delta f}{f} \quad \text{Equation 1}$$

This equation gives us the wavelength shift of light due to a moving source or a moving observer. The fractional change in wavelength, $\frac{\Delta\lambda}{\lambda}$, (or frequency, $\frac{\Delta f}{f}$) is just the ratio of the source's speed to the speed of light, $\frac{v}{c}$. So, if a light source is moving at 1% of the speed of light, we get a 1% change in both the wavelength and the frequency of the light. This equation also applies to observers moving away or toward a light source and starts to get particularly exciting when the observer just happens to be an atom.

3

As an example, we can take a calcium atom in an ideal gas of calcium atoms at 300 K. So, the rms speed of a calcium atom can be calculated from the kinetic theory.

4

$$\frac{1}{2} mc^2 = \frac{3}{2} kT \quad \text{Equation 2}$$

Knowing that the mass of a calcium atom is **40 u**, you get a speed of around 400 m s^{-1} . This means that when light approaches a calcium atom head-on, the calcium atom typically sees the light as being blue-shifted by a fraction $\frac{400}{3 \times 10^8}$.

5

Another important thing we just happen to know about calcium is that it has a red line in its emission (and absorption) spectrum. The frequency of this red line is $456\,000.6 \text{ GHz}$. We will then calculate the blue-shifted frequency that a 400 m s^{-1} calcium atom sees when the atom has a head-on collision with light of this frequency.

6

$$\Delta f = \frac{400}{3 \times 10^8} \times f \approx 0.6 \text{ GHz} \quad \text{Equation 3}$$

So, when a calcium atom has a head-on collision with light of frequency $456\,000.6 \text{ GHz}$, the calcium atom says that the frequency of the light is $456\,001.2 \text{ GHz}$. It might seem that such a tiny difference would be completely unimportant. However, this difference is enough to stop the atom from absorbing the light. The light passes straight through the atom and no momentum is transferred to the atom. But when the frequency of the light is exactly right, you can get absorption. To get this frequency exactly right we use light of frequency $456\,000.0 \text{ GHz}$. Then, when the calcium atom has a head-on collision with this light it says that the frequency is:

7

$$456\,000.0 \text{ GHz} + 0.6 \text{ GHz} = 456\,000.6 \text{ GHz}.$$

This is the exact frequency for a calcium atom to absorb a photon of our light.

8



So how do we actually do the cooling? First, we shine light of frequency 456 000.0 GHz at our calcium gas and remember that only head-on collisions will lead to absorption of photons. Then, you have to remember that photons have momentum and absorbing a head-on photon will slow down the atom which means that the gas has cooled. The momentum of a photon is given by the following equation: 9

$$p = \frac{h}{\lambda} \quad \text{Equation 4}$$

So how much does one photon slow down a calcium atom? Conservation of momentum tells us that it's a rather disappointing 1.5 cm s^{-1} . What?? All that theory and all we did was slow down a calcium atom by 1.5 cm s^{-1} ??? But then you have to think to yourself, "What happens if we can persuade the calcium atom to absorb 26 000 photons?" 10
Fortunately for us, a 50 mW laser tends to send out a lot of photons so that absorbing 26 000 photons only takes about a millisecond.

But what happens if a photon comes from behind and tries to accelerate our atoms? An important thing to remember about this technique is that if the atom is hit from behind by the photon, the atom will see the light as being red-shifted. This means that the frequency has changed the wrong way and the photon will not be absorbed (it no longer fits our exact frequency requirement). So, our atoms will only be slowed down and never accelerated as long as we choose the frequency of light carefully. 11

Are there any problems that have to be overcome? Of course, there are, otherwise I wouldn't be asking the question: 12

1. What happens to the excited atom when it **emits** a photon? Doesn't it gain the momentum it just lost? The answer to that is "No". When atoms emit photons, they do so in random directions. These photons, on average, do not change the rms speed of the gas because the photon has a 50:50 chance of being emitted with a velocity component in the direction of motion of the atom or opposite to the direction of motion. However, the recoil of a calcium atom after emitting a photon means that you can't really slow the atoms below around 1.5 cm s^{-1} . 13
2. How do we stop the atoms from hitting the sides of the container? The answer is that we use a magnetic trap – it even works on neutral atoms!! 14
3. Does the frequency of the laser have to change as the atoms slow down? Yes, as the atoms slow down, the blue-shift decreases and we have to use frequencies that are closer to 456 000.6 GHz. We can actually do this by oscillating our laser mirrors using piezoelectric crystals. The frequency change of the laser is equal to the frequency of oscillation of the mirrors. 15

Finally, does this actually work or is it just another thought experiment that works only in Einstein's brain (if he were still alive)? The answer to that question is "Yes, since 1978!!!" 16
Furthermore, temperatures as low as 1 pK may be reachable using the Cold Atom Laboratory which is already on board the International Space Station.



Answer the following questions in your own words. Direct quotes from the original article will not be awarded marks.

- (a) Explain why there is a negative sign in Equation 1. [2]

$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = -\frac{\Delta f}{f}$$

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- (b) (i) Explain why a calcium atom travelling at 400 ms^{-1} will absorb a photon coming directly toward it when the frequency of the photon is $456\,000.0 \text{ GHz}$ (see paragraphs 6–8). [2]

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- (ii) Explain why a calcium atom travelling at 400 ms^{-1} will not absorb a $456\,000.0 \text{ GHz}$ photon if the photon and atom are moving in the same direction (see paragraphs 6–8 and 11). [1]

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- (iii) Hence, explain very briefly how calcium atoms can be cooled using light of this frequency (see paragraphs 6–11). [1]

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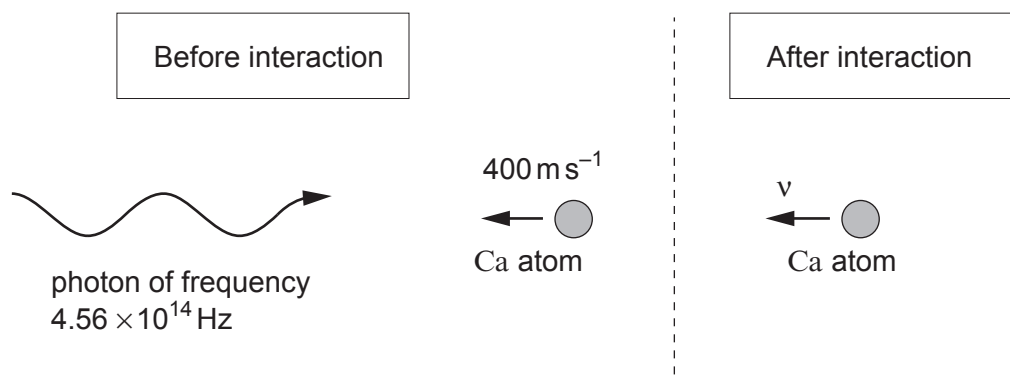
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- (c) Helium has a rms speed of $1\,350\text{ m s}^{-1}$ and an emission line of **wavelength** 587.56148 nm . Calculate the best laser wavelength for slowing down helium atoms. Give your answer to 8 significant figures (see Equation 1 and paragraphs 6–11). [4]

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- (d) Show that, when a calcium atom of mass 40 u absorbs a photon of frequency $4.56 \times 10^{14}\text{ Hz}$, its velocity decreases by approximately 1.5 cm s^{-1} . [4]



TURN OVER FOR THE LAST PARTS OF THE QUESTION



- (e) The author states that a calcium atom will absorb 26 000 photons in about a millisecond. Discuss whether or not this is possible, given the power of the laser and given that the lifetime of the excited calcium atom is a few nanoseconds. You should reinforce your answer with calculations (see paragraph 10). [3]

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- (f) Rhian claims that the random re-emission of photons by excited atoms means that the lowest temperature achievable by this method is approximately $0.1 \mu\text{K}$. Determine whether she is correct (see Equation 2 and paragraphs 10 and 13). [3]

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END OF PAPER



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GCE A LEVEL

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THURSDAY, 26 MAY 2022 – AFTERNOON

PHYSICS – A level component 1

Data Booklet

A clean copy of this booklet should be issued to candidates for their use during each A level component 1 Physics examination.

Centres are asked to issue this booklet to candidates at the start of the course to enable them to become familiar with its contents and layout.

Values and Conversions

Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Fundamental electronic charge	$e = 1.60 \times 10^{-19} \text{ C}$
Mass of an electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Molar gas constant	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Acceleration due to gravity at sea level	$g = 9.81 \text{ ms}^{-2}$
Gravitational field strength at sea level	$g = 9.81 \text{ N kg}^{-1}$
Universal constant of gravitation	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Planck constant	$h = 6.63 \times 10^{-34} \text{ Js}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Speed of light in vacuo	$c = 3.00 \times 10^8 \text{ ms}^{-1}$
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$
Stefan constant	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Wien constant	$W = 2.90 \times 10^{-3} \text{ m K}$
Hubble constant	$H_0 = 2.20 \times 10^{-18} \text{ s}^{-1}$

$$T/\text{K} = \theta/^{\circ}\text{C} + 273.15$$

$$1 \text{ parsec} = 3.09 \times 10^{16} \text{ m}$$

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$\frac{1}{4\pi\epsilon_0} \approx 9.0 \times 10^9 \text{ F}^{-1} \text{ m}$$

$\rho = \frac{m}{V}$	$T = 2\pi\sqrt{\frac{L}{g}}$
$v = u + at$	$pV = nRT$ and $pV = NkT$
$x = \frac{1}{2}(u + v)t$	$p = \frac{1}{3}\rho\overline{c^2} = \frac{1}{3}\frac{N}{V}m\overline{c^2}$
$x = ut + \frac{1}{2}at^2$	$M / \text{kg} = \frac{M_r}{1000}$
$v^2 = u^2 + 2ax$	$n = \frac{\text{total mass}}{\text{molar mass}}$
$\Sigma F = ma$	$k = \frac{R}{N_A}$
$p = mv$	$U = \frac{3}{2}nRT = \frac{3}{2}NkT$
$W = Fx\cos\theta$	$W = p\Delta V$
$\Delta E = mg\Delta h$	$\Delta U = Q - W$
$E = \frac{1}{2}kx^2$	$Q = mc\Delta\theta$
$E = \frac{1}{2}mv^2$	$I = \frac{\Delta Q}{\Delta t}$
$Fx = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	$I = nAve$
$P = \frac{W}{t} = \frac{\Delta E}{t}$	$R = \frac{V}{I}$
$\text{efficiency} = \frac{\text{useful energy transfer}}{\text{total energy input}} \times 100\%$	$P = IV = I^2R = \frac{V^2}{R}$
$\omega = \frac{\theta}{t}$	$R = \frac{\rho l}{A}$
$v = \omega r$	$V = E - Ir$
$a = \omega^2 r$	$\frac{V}{V_{\text{total}}} \left[\text{or } \frac{V_{\text{OUT}}}{V_{\text{IN}}} \right] = \frac{R}{R_{\text{total}}}$
$a = \frac{v^2}{r}$	$C = \frac{Q}{V}$
$F = \frac{mv^2}{r}$	$C = \frac{\epsilon_0 A}{d}$
$F = m\omega^2 r$	$E = \frac{V}{d}$
$a = -\omega^2 x$	$U = \frac{1}{2}QV$
$x = A\cos(\omega t + \epsilon)$	$Q = Q_0 \left(1 - e^{-\frac{t}{RC}} \right)$
$T = \frac{2\pi}{\omega}$	$Q = Q_0 e^{-\frac{t}{RC}}$
$v = -A\omega\sin(\omega t + \epsilon)$	$F = kx$
$T = 2\pi\sqrt{\frac{m}{k}}$	$\sigma = \frac{F}{A}$

$\varepsilon = \frac{\Delta l}{l}$	$n = \frac{c}{v}$																				
$E = \frac{\sigma}{\varepsilon}$	$n_1 v_1 = n_2 v_2$																				
$W = \frac{1}{2} Fx$	$n_1 \sin \theta_1 = n_2 \sin \theta_2$																				
$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$	$n_1 \sin \theta_C = n_2$																				
$F = G \frac{M_1 M_2}{r^2}$	$E_{\text{k max}} = hf - \phi$																				
$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$	$p = \frac{h}{\lambda}$																				
$g = \frac{GM}{r^2}$	$A = \lambda N$																				
$V_E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$	$N = N_0 e^{-\lambda t}$																				
$\text{PE} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r}$	$A = A_0 e^{-\lambda t}$																				
$V_g = -\frac{GM}{r}$	$N = \frac{N_0}{2^x}$																				
$\text{PE} = -\frac{GM_1 M_2}{r}$	$A = \frac{A_0}{2^x}$																				
$W = q\Delta V_E$	$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$																				
$W = m\Delta V_g$	<table><tr><td></td><th colspan="2">leptons</th><th colspan="2">quarks</th></tr><tr><th>particle (symbol)</th><th>electron (e⁻)</th><th>electron neutrino (ν_e)</th><th>up (u)</th><th>down (d)</th></tr><tr><th>charge (e)</th><td>- 1</td><td>0</td><td>+$\frac{2}{3}$</td><td>-$\frac{1}{3}$</td></tr><tr><th>lepton number</th><td>1</td><td>1</td><td>0</td><td>0</td></tr></table>		leptons		quarks		particle (symbol)	electron (e ⁻)	electron neutrino (ν _e)	up (u)	down (d)	charge (e)	- 1	0	+ $\frac{2}{3}$	- $\frac{1}{3}$	lepton number	1	1	0	0
		leptons		quarks																	
particle (symbol)		electron (e ⁻)	electron neutrino (ν _e)	up (u)	down (d)																
charge (e)		- 1	0	+ $\frac{2}{3}$	- $\frac{1}{3}$																
lepton number		1	1	0	0																
$\lambda_{\text{max}} = \frac{W}{T}$																					
$P = A\sigma T^4$																					
$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$																					
$v = H_0 D$																					
$\rho_c = \frac{3H_0^2}{8\pi G}$	$E = mc^2$																				
$r_1 = \frac{M_2}{M_1 + M_2} d$	$F = BIl \sin \theta$																				
$T = 2\pi \sqrt{\frac{d^3}{G(M_1 + M_2)}}$	$F = Bqv \sin \theta$																				
$T = \frac{1}{f}$	$B = \frac{\mu_0 I}{2\pi a}$																				
$c = f\lambda$	$B = \mu_0 nI$																				
$\lambda = \frac{a\Delta y}{D}$	$\Phi = AB \cos \theta$																				
$d \sin \theta = n\lambda$	flux linkage = $N\Phi$																				

Mathematical Information

SI multipliers

Multiple	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c

Multiple	Prefix	Symbol
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z

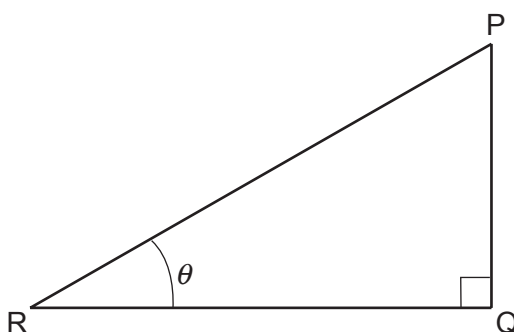
Areas and Volumes

$$\text{Area of a circle} = \pi r^2 = \frac{\pi d^2}{4}$$

$$\text{Area of a triangle} = \frac{1}{2} \text{ base} \times \text{height}$$

Solid	Surface area	Volume
rectangular block	$2(lh + hb + lb)$	lbh
cylinder	$2\pi r(r + h)$	$\pi r^2 h$
sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Trigonometry



$$\sin \theta = \frac{PQ}{PR}, \quad \cos \theta = \frac{QR}{PR}, \quad \tan \theta = \frac{PQ}{QR}, \quad \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$PR^2 = PQ^2 + QR^2$$

Logarithms

[Unless otherwise specified 'log' can be \log_e (i.e. \ln) or \log_{10} .]

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log x^n = n \log x$$

$$\log_e e^{kx} = \ln e^{kx} = kx$$

$$\log_e 2 = \ln 2 = 0.693$$